Reduction of Complexity: An Aspect of Network Visualization

Jan Terje Bjørke
Norwegian Defence Research Establishment
P O Box 115
NO-3191 Horten
Norway

email: jtb@ffi.no

ABSTRACT

Networks are topological structures composed of nodes and arcs. Often, networks are visualized by point symbols and lines illustrating the nodes and the arcs, respectively. As the number of the nodes and links increases, the visual representation of the network needs generalization in order to keep the visual clarity of the image. To solve the problem considered, a methodology to aggregate nodes and links into hyper-nodes and hyper-links is developed. The algorithm, which is based on an information theoretic approach to reorder the adjacency matrix of the network, can generate hierarchies of hyper-networks. This kind of generalization algorithm can be used to construct images which visualize the main structures of the network. Some case studies demonstrate the algorithm.

1 INTRODUCTION

Networks are topological structures composed of nodes and arcs. Often, networks are visualized by point symbols and lines illustrating the nodes and the arcs, respectively. These simple visualization techniques need to be refined when complex real world phenomenon are to be visualized. For example, we can ask how to deal with the possible visual overload of nodes and arcs, or how to illustrate uncertainty or fuzziness in the information. The purpose of the present research is to identify strategies for the visualization of network information.

Distinction can be made between visual communication and visual exploration (MacEachern 1994). Visual communication deals with how to visualize results of different kinds of analysis, i.e., visualization in the case that the message is well defined. Another view is covered by visual exploration. In this case the message is not well defined and the analysis of structure or detection of important information is left to the map reader, i.e., exploration is permitted by the provision of various tools.

The reduction of complexity is a clue in the visualization of networks with many nodes and links. Therefore, I have developed a methodology to aggregate nodes and links into hyper-nodes and hyper-links. The algorithm, which is based on an information theoretic approach to reorder the adjacency matrix of the network, can generate hierarchies of hyper-networks. This kind of generalization can be used to solve the problem of visual overload in the presentation and enhance the understanding of complex networks. Some case studies will demonstrate the algorithm.

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2 REVIEW OF LITTERATURE

Fabrikant et al. (2004) show how spatialization is used to generate an information display of non-spatial data. They point out that information spatialization is inspired by the intuition that spatial displays as maps, charts or diagrams can help to amplify cognition. Spatialization typically rely on dimension reduction techniques and layout algorithms to project relatedness in non-spatial data content onto distance, such that semantically similar documents are placed closer to one other than less similar ones in an information space. Their empirical study suggests that the distance-similarity metaphor applies to network spatializations by equating metric distance along network lines to similarity. They also find that line size, colour value and hue, modify the distance-similarity metaphor in subtle yet logical ways. The idea of mapping nodes into hyper-nodes can be seen as an extreme variant of mapping relatedness onto distance.

Etien et al. (2004) present an approach that combines visual and computational analysis to deal with large volumes of geospatial data. Their approach is based on the application of computational algorithms to extract patterns and relationships in the data. The visual representations are applied to portray the extracted patterns. The idea of constructing hierarchies of hyper-graphs, is a kind of pattern extraction.

Lai et al. (2004) investigates the effectiveness of dynamic symbols in cartographic communication. Their experiments confirmed that dynamic symbols tend to attract the attention of users and even in situations when foreground-background contrast is poor. On the question of how many flashing or animated symbols a display could sustain before the flickering became distracting, they conclude that 15 blinking symbols on a single display were acceptable. Flashing symbols will not be used in the forthcoming visualization algorithm. In order to add information to hyper-nodes and hyper-links, I have applied the visual variable size, where the size is determined from the number of the underlying nodes and arcs.

Perceptual conflicts in the visual representation of networks can be solved by different cartographic generalization operators. Bjørke and Isaksen (2005) show how information theory can be used to select an optimal number of arcs in a network. Their methodology is demonstrated on road networks in Norway. Harrie (1999 and 2003) shows how the least squares method can be applied to move map objects from its original position to a new in order to reduce the amount of visual conflicts. This method is certainly also applicable to the visualization of networks. Højholt (2000) presents a similar displacement algorithm based on the finite element method. In the case that the arcs of the network not are straight lines, but are sinuous, line smoothing techniques can be applied to increase their visual clarity. Different methods can be applied from the simple ones to more sophisticated techniques based on energy minimization (Burghardt 2005), for example.

MacEachern et al. (2005) give a review of visualization of uncertainty in geographical information and identify key research challenges in the field considered. Slocum et al. (2003) developed a tool for a wall-size display to visualize the global water balance together with the uncertainties associated with the model. They report that all the participants in their investigations were enamoured with the wall-size display, but to prove its effectiveness compared with more traditional displays, needs more investigations. An interesting aspect of their investigation is that decision makers are likely to feel uncomfortable with the notion of uncertainty. This fact implies that we should not merely present uncertainties, but also suggest how to deal with uncertainty.

Plewe (2003) demonstrates how various sources produce uncertainties that are different in their nature and in their appearance. He recognizes two types of uncertainty, ambiguity and fuzziness (Klir and Wierman 1998, p.105). Ambiguity is an acknowledgement that an observation is only one of many possible measurable values for a phenomenon. In contrast, fuzziness is an inherent characteristic of the phenomenon being observed. Plewe (2003) points out that ambiguity and fuzziness need to be treated as separate for three reasons: (1) those using the data must be able to differentiate them, (2) both types might be present simultaneously, and (3) one type can be transformed into the other type.

3 INFORMATION THEORY APPLIED TO THE EVALUATION OF MAP COMPLEXITY

A perception study was carried out in order to find the optimal number of colored depth intervals in seafloor maps. Approximately, twenty five subjects participated in the study. From the study we derived the conditional probabilities that the colored intervals were misinterpreted. Then the difference between the map entropy and the equivocation was computed. The map with seven intervals on the scale from light to dark blue did correspond to the channel capacity. One can ask to which degree this experiment can be generalized to network visualization. Are seven nodes optimal? Probably not in general, since this kind of generalization is too simplistic.

Miller (1956) presents several investigations where an information theoretic approach is used to measure the perceptual performance of subjects. In his experiments the number of seven separable categories was certified for one dimensional stimulus, like the scale of blue colours from light to dark. Since a map is embedded in a two-dimensional plane, we can argue that the number of seven from the one-dimensional case should be increased. A deeper understanding of this topic is outside the scope of the present paper, but it is an item for further research.

The recognition that there is an optimal number of nodes leads to the concept of hyper-networks. Each hyper-node consists of several sub-nodes and each hyper-link is composed of sub-links. We can imagine a zoom into the hyper-network as well as its hyper-nodes. How to derive this structure can follow to principles: (1) the communication view and (2) the exploration view. In the first case the structure is predefined to the user. In the latter the structure can be generated and manipulated by the map reader. The communication view can be realized as series of predefined maps. The exploratory view requires some kind of dynamics, i.e., the user must be able to manipulate the visualization of the network.

4 METHOD TO DERIVE HYPER-NETWORKS

Bertin (1981) shows how the reorderable matrix can be used to find categories or groups in geographical data. This technique is termed seriation. Bjørke and Smith (1996) present an algorithm to automate the seriation of a reorderable matrix. They define the seriation criterion on the basis of the minimum entropy of a binary image. This method can be applied to the definition of hyper-nodes in a network. The reorderable matrix in this case, is the adjacency matrix of the network. This matrix can easily be mapped to a binary image and the method proposed by Bjørke and Smith (1996) can be applied to reorder the matrix.

Figures 1 to 2 demonstrate the method. The adjacency matrix in Figure 1 is reordered as shown in Figure 2. From the reordered matrix, three groups of nodes are derived, i.e., the hyper-nodes h1, h2 and h3 as shown in Figure 2. From the hyper-network we can see that the initial network in Figure 1 can be regarded as a tree structure, one mother node and two leave nodes. The mother node is composed of three sub-nodes (1,4 and 5) which have strong connection, i.e., one node is connected to the other two. There is no direct connection between the leave nodes.

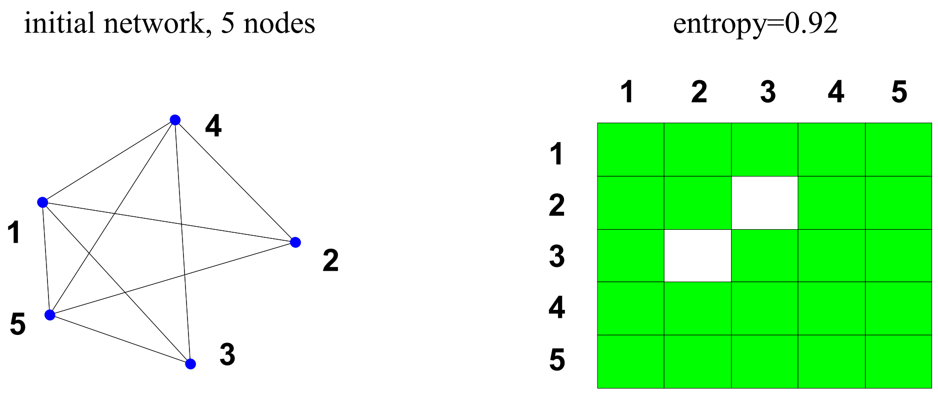

Figure 1: A network and its adjacency matrix

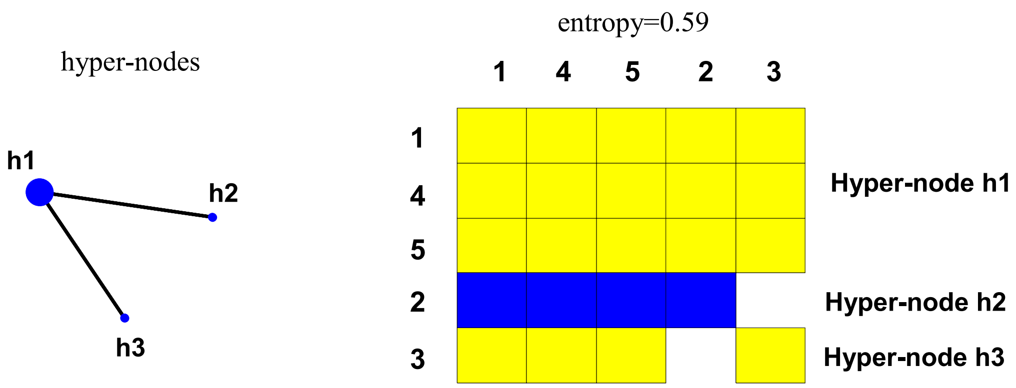

Figure 2: The matrix in Figure 1 after reordering. The hyper-nodes in the left window are derived from grouping the rows of the matrix. The group factor is set to 1.0.

Grouping of the nodes require a similarity measure. An index on the sale [0,1] measures the minimum similarity for the rows in a group. If the group factor is 1, the rows must be completely similar in order to be mapped to the same group. In the example in Figure 2 the factor is set to 1. Since the group factor determines the aggregation of the nodes, it should be user defined.

In order to visualize the number of sub-nodes of a hyper-node, the visual variable size is used. From Figure 2 we can se that the hyper-node with three sub-nodes, i.e., h1, is marked with a larger circle than the other two nodes. Similarly, the line thickness illustrates the number of sub-links of a hyper-link.

Another example of the reordering of the adjacency matrix is shown in Figures 3-4. From Figure 4 we can see the hyper-nodes derived from the reordered matrix. In this case the group factor is set to 0.5, i.e., we have loosened the requirement for the connection between the sub-nodes of a hyper-node. In the previous example the group factor was set to one.

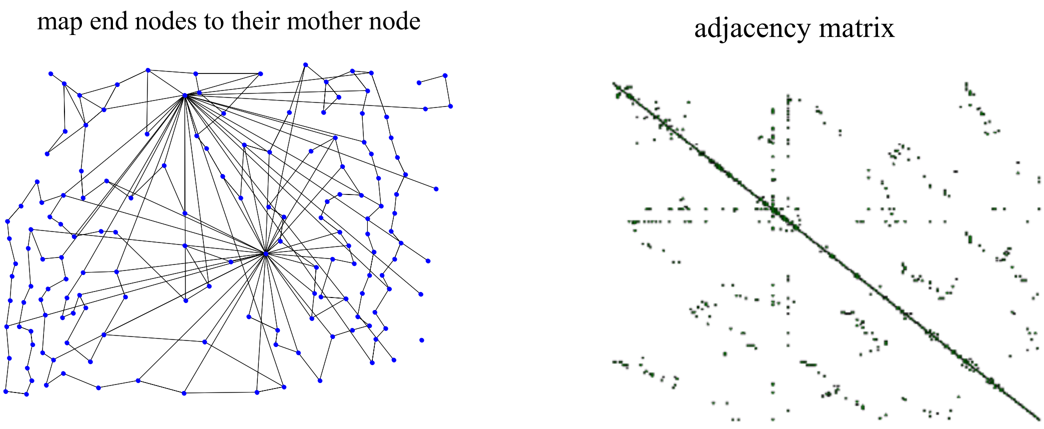

Figure 3: A network and its adjacency matrix

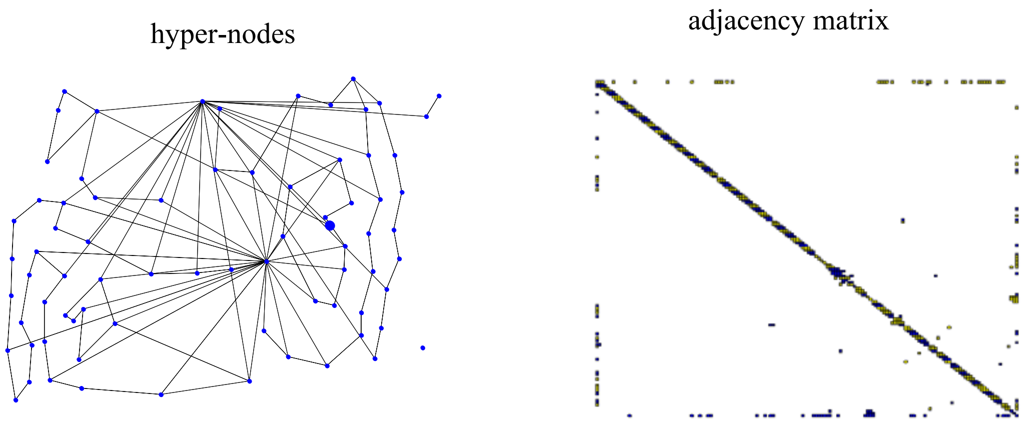

Figure 4: The matrix in Figure 3 after reordering. The hyper-nodes in the left window are derived from grouping the rows of the matrix.

When mapping the hyper-nodes to a position in the plane, we must decide where to put the nodes. In the application presented, the (x,y) co-ordinates of the hyper-nodes are computed as the average position of its sub-nodes. This choice is questionable if dealing with networks embedded in a geographical space. Therefore, the location of the hyper-nodes is a topic for further research.

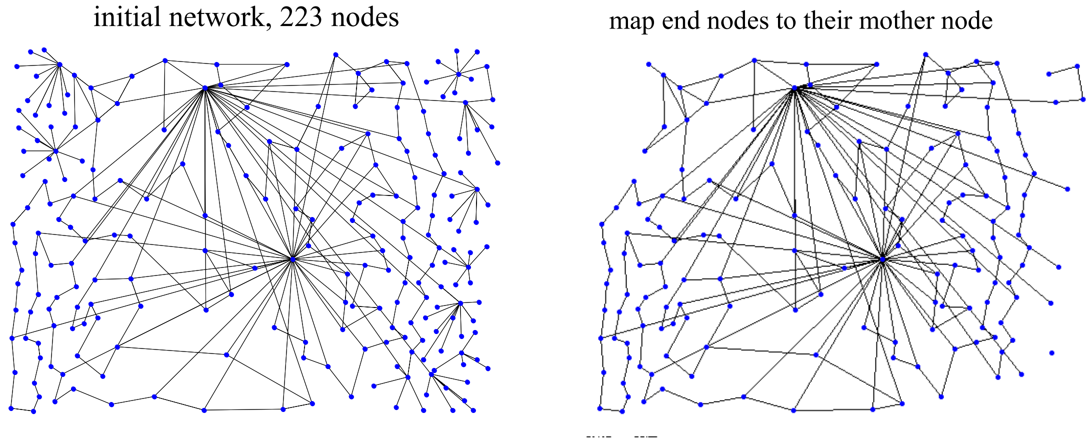

Figure 5: An initial network with many end nodes and the result after mapping these nodes to their mother node.

Figure 5 illustrates a network with many end nodes and how this network can be simplified. Conceptually, I will claim that an end-node has so strong relations to its mother node that end nodes can be mapped to their mother node, as shown in the figure. This mapping can be run before the seriation algorithm is started.

The hyper-network algorithm is demonstrated in Figure 6 by a sequence of hyper-networks. Image 1 shows the initial network. In image 2 the end nodes are mapped to their mother node. The disconnected sub-network C is hardly seen in image 1, but image 2 clarifies the component considered. This demonstrates how the clutter of the lines in image 1 hides that C is disconnected from the rest of the network. Then the seriation algorithm is started. The result is shown in image 3.

The recursive nature of the proposed algorithm is clarified in image 4. Here, the hyper-network is derived from applying the seriation algorithm to the network in image 3. Thereafter, the algorithm is applied to image 4 and so on until image 8 is reached. Image 8 shows the initial network as one large hyper-node and the small disconnected node C. In image 7 the hierarchical property of the initial network is illustrated. Here, node A appears as a root node in the network. Image 7 also illustrates how the circle size and the line thickness are associated to the number of sub-nodes and sub-links, respectively.

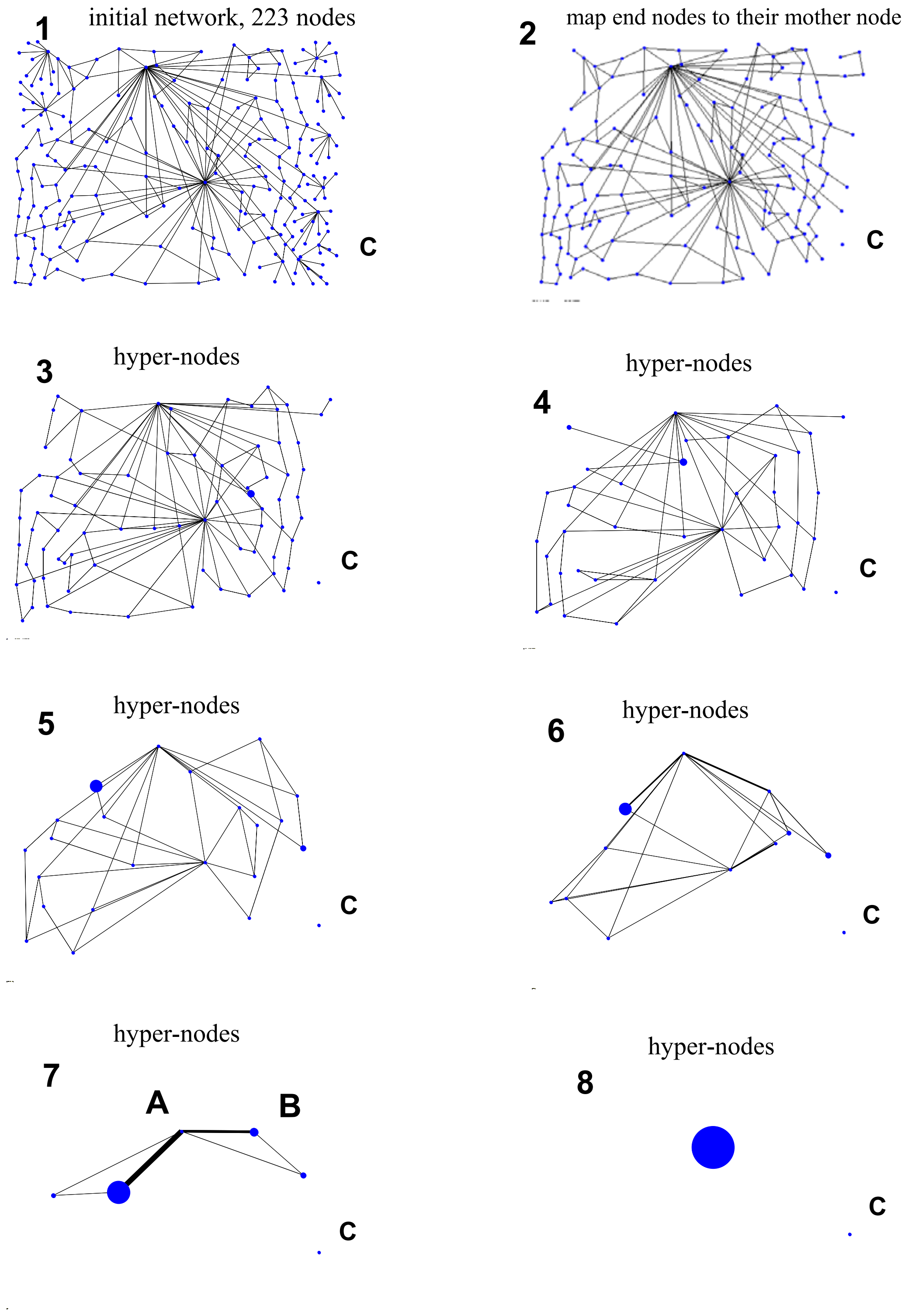

Figure 6: A hierarchy of hyper-networks. The group factor is set to 0.5.

One can ask what happens to the connectivity of the network if the root node A in image 7 is eliminated. This kind of functionality is implemented into the software I have developed. The effect of eliminating hyper-node A is illustrated in Figure 7. The eliminated node has four sub-nodes. After the elimination the initial network is reduced to the network as illustrated in the left image of Figure 7. The reduced network is decomposed into twelve disconnected sub-networks. The decomposition is clearly seen in the window to the right, which is derived from applying the seriation algorithm to the reduced network. The eliminated nodes are shown in red.

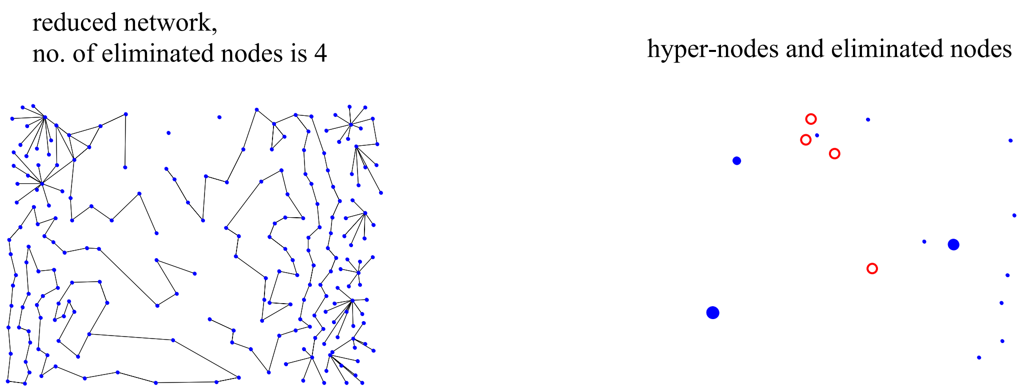

Figure 7: The effect of eliminating hyper-node A Figure 6. The node to be eliminated appears as a root node in image 7 of Figure 6. After elimination the initial network is decomposed into twelve disconnected networks. The decomposition is clearly seen in the window to the right. The eliminated nodes are shown in red.

The next question is what happens to the network if a hyper-link is eliminated. This kind of manipulation of a network is demonstrated in Figure 8. The network in Figure 6 is used for the illustration. Hyper-link A-B in image 7 of Figure 6 is eliminated. Twenty one sub-arcs are associated to the hyper-link considered. The result of the elimination is shown in Figure 8. Image 1 shows the initial network after the elimination. The reduced network is still connected, except three nodes in the upper part of the graph. These nodes, termed a, b and c, stands clearly out in the hyper-network in image 2. The disconnected nodes can hardly be seen in image 1. Image 3 shows the eliminated arcs, and finally in image 4 is illustrated the hyper-network of the top of the hierarchy of the generalized networks.

5 CONCLUSIONS

According to information theory there exists an optimal number of nodes in the visualization of networks. This principle leads to the theory of reduction of map complexity. I have not tried to compute the optimum number of nodes in network visualizations, but have proposed a methodology to zoom in and out of the network. This method offers the user a tool to control the map complexity. In this way the network analyst will get an impression of the main structures of the network.

A reordering technique of the adjacency matrix of the network is applied. In this way a hierarchy of hyper-networks can be derived. The reordering of the matrix is formulated on the basis of minimum entropy criteria. In addition to the construction of hyper-networks, tools to study the effect of elimination of hyper-nodes and hyper-links are implemented. This kind of manipulation of the network is demonstrated.

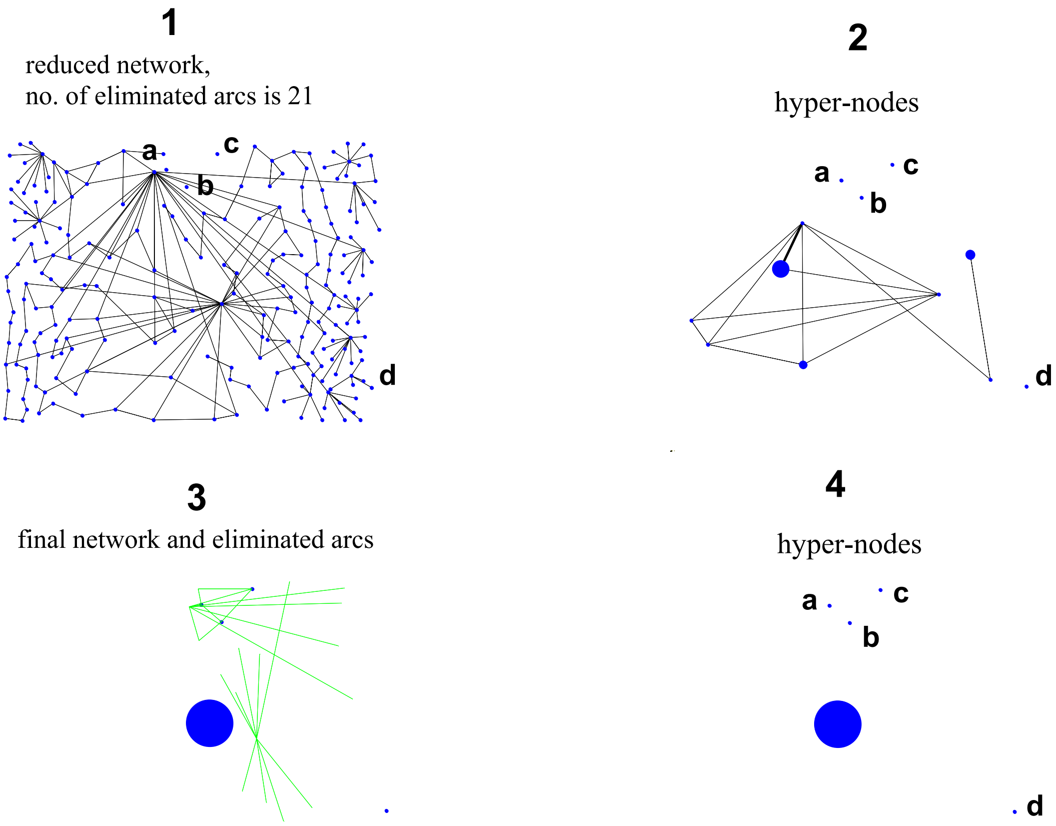

Figure 8: The effect of eliminating a hyper-link between the nodes A and B in image 7 of Figure 6. The eliminated arc has 21 sub-arcs. After elimination of these arcs, the reduced network is still connected, except three nodes in the upper part of the graph. The eliminated arcs are shown in green in image 3.

The group factor which is I have implemented, can be manipulated by the user. Automatic derivation of an optimal parameter value requires further study. Another topic for further research is how to compute the position of the hyper-nodes. This can become an urgent problem when the network is to be embedded in a geographical space. Also further development of information theoretic views is an interesting research direction, for example, the integration of the hyper-network method and methods for arc elimination (Bjørke and Isaksen, 2005).

REFERENCES

Bertin, J. (1981). *Graphics and graphic information processing*. Walter de Greuter: Berlin, 273 pp.

Bjørke, J. T. and E. Isaksen (2005). Map Generalization of Road Networks: Case study from Norwegian small scale maps. Proceedings *XXII International cartographic Conference*, A Coruña, Spain, 11-16 July 2005.

Bjørke, J. T. and B. Smith (1996). Seriation: an implementation and case study. *Computers, Environment and Urban Systems,* 20(6), 427-438.

Burghardt, D. (2005). Controlled line smoothing by snakes. *Geoinformatica* 9:3, 237-252.

Etien, L. K. and M.-J. Kraak (2004). Alternative visualization of large geospatial datasets. *Cartographic Journal,* 41:3, 217-228.

Fabrikant, S. I., D. R. Montello, M. Ruocco, and R. S. Middleton (2004). The distance-similarity metaphor in network-display spatializations. *Cartography and Geographic Information Science,* 31:4, 237-252.

Harrie, L. (2003). Weight-setting and quality assessment in simultaneous graphic generalization. *The Cartographic Journal,* 40:3, 221-233.

Harrie, L. (1999). The constrained method for solving spatial conflicts in cartographic generalization. *Cartography and Geographic Information Science*, 26, 55-69.

Højholt, P. (2000). Solving space conflicts in map generalization: using a finite element method. *Cartography and Geographic Information Science*, 27, 65-73.

Klir, G. J. and M. J. Wierman (1998). *Uncertainty-based information: elements of generalized information theory.* Heidelberg, Germany: Physica-Verlag.

Lai, P.C. and A. Gar-On Yeh (2004). Assessing the effectiveness of dynamic symbols in cartographic communication. *Cartographic Journal*, 41:3, 229-244.

MacEachren, A. M. (1994). Visualization in modern cartography: setting the agenda, *in*, MacEachren, A. M., and Taylor, D. R. F., Visualization in Modern Cartography, Pergamon: Oxford, p. 1-12.

Miller, G. A. (1956). The magical number of seven, plus minus two: some limits on our capacity for processing information. *The Psycological Review,* Vol. 63, no. 2, p. 81-97.